Musik in Bayern

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)

https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

Oscillation for a Class of Variable order nonlinear Fractional Differential Equation

Dr.B.Rani

Department of Mathematics, Sri Kanyaka Parameswari Arts & Science College for Women, Chennai-1, Tamilnadu, India. Email: raninagendran2002@gmail.com

Abstract: In this article, we are concerned with oscillation of all solutions for a class of variable order fractional differential equation of the type $a(t)g(D_{-}^{\beta(t)}Y)(t)]' - b(t)\, f\left(\int_t^{\infty}(z-t)^{-\beta(t)}Y(z)dz\right) = 0$, for $t > 0$ where $0 < \beta(t) < 1$ and $\left(D_{-}^{\beta(t)}Y\right)$ is the Liouville right-sided fractional derivative of order $\beta(t)$ of Y. Using the generalized Riccati transformation technique,we have figured out the oscillation criteria for a certain type of variable order nonlinear fractional differential equation and a few applications are shared to highlight the results we have established.

Keywords: Class of variable order fractional differential equation, oscillation, fractional derivative, fractional integral, Riccati techniques.

1. Introduction

Fractional differential equations hold considerable importance across numerous scientific and engineering disciplines, such as electrical networks, control theory, and viscoelasticity. They possess extensive application in physics, engineering, biology, and finance for modeling memory and hereditary characteristics.There are quite a few books out there that dive into fractional derivatives and integrals, like the ones from references [1-5].

The oscillation of fractional differential equations was examined in [9–16].

In [18], the author explored the oscillatory behavior of a class of fractional differential equation with damping.

$$(D_{-}^{1+\alpha}y)(t) - p(t)\,(D_{-}^{\alpha}y)(t) + q(t)\, f\left(\int_t^{\infty}(v-t)^{-\alpha}\, y(v)dv\right) = 0, \text{ for } t > 0 \qquad (1.1)$$

Where $D_{-}^{\alpha}y$ is the Liouville right-sided fractional derivative of order $\alpha \in (0, 1)$ of y.

In [19], the authors discussed the oscillatory solutions for a class of the fractional differential equation.

$$[r(t)\, g(D_{-}^{\alpha}y)(t)]' - p(t)\, f\left(\int_t^{\infty}(s-t)^{-\alpha}\, y(s)ds\right) = 0, \text{ for } t > 0 \qquad (1.2)$$

where $0 < \alpha < 1$ is a real number, $D_{-}^{\alpha}y$ is the Liouville right-sided fractional derivative of order α of y, r and p are positive continuous functions on $[t_0, \infty)$ for $t_0 > 0$.

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)

https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

In this article, we study the oscillatory criteria for the variable order fractional differential equation with the type

$$[a(t)\, g(D_{-}^{\beta(t)}Y)(t)]' - b(t)\ f\left(\int_t^{\infty}(z-t)^{-\beta(t)}\ Y(z)\ dz\right) = 0 \ , \text{ for } t > 0 \qquad (1.3)$$

where $0 < \beta(t) < 1$, $(D_{-}^{\beta(t)}Y)$ is the Liouville right-sided fractional derivative of order $\beta(t)$ of Y defined by

$$(D_{-}^{\beta(t)}Y)(t) = -\left(\frac{1}{\Gamma_{(1-\beta(t))}}\right)\frac{d}{dt}\int_t^{\infty}(z-t)^{-\beta(t)}\ Y(z)\ dz \text{ for } t \in R^{+} := (0, \infty),$$

here $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(t) = \int_t^{\infty} e^{-z}\, z^{t-1}\, dz$ for $t \in R^{+}$, and the following conditions are assumed to hold:

(A_1) a and b are two nonnegative continuous functions on $t \in [t_0, \infty)$ for $t_0 > 0$;

(A_2) $g^{-1} \in C(R, R)$ are continuous function with $pg^{-1}(p) > 0$ for $p \neq 0$, and $\exists$ some positive constant α such that

$g^{-1}(pq) \geq \alpha\, g^{-1}(p)\, g^{-1}(q)$ for $pq \neq 0$;

(A_3) $f, g : R \to R$ are continuous function with $uf(u) > 0$, $ug(u) > 0$ for $u \neq 0$, and $\exists$ positive constants ℓ_1, ℓ_2 such that

$f(u)/u \geq \ell_1$, $u/g(u) \geq \ell_2$ for all $u \neq 0$.

By a solution of (1.3), we mean a nontrivial function $Y \in C(R^{+}, R)$ with $\int_t^{\infty}(z-t)^{-\beta(t)}\ Y(z)\ dz \in C^1(R^{+}, R)$ and $a(t)\ g(D_{-}^{\beta(t)}Y)(t) \in C^1(R^{+}, R)$ satisfies (1.3) for $t > 0$. Our attention is restricted to those solutions of (1.3) which exist on R^{+} and satisfy $\sup\{|Y(t)| : t > t_*\} > 0$ for any $t_* \geq 0$. A solution Y of (1.3) is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise it is nonoscillatory.

2. Preliminaries

Definition 2.1: A solution of a differential equation is said to be oscillatory if it has arbitrarily many zeros. If all the solutions of an equation

are oscillatory, then the differential equation is said to be oscillatory.

Definition 2.2 The Liouville right-sided fractional integral of order $\beta(t) > 0$ of a function $f : R^{+} \to R$ on the half-axis R^{+} is given by

$$(I_{-}^{\beta(t)}f)(t) = \frac{1}{\Gamma_{(\beta(t))}}\int_t^{\infty}(z-t)^{\beta(t)-1}\ f(z)\ dz, \text{ for } t > 0 \qquad (2.1)$$

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)
https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

Provided that the right side is point wise defined on R^+, where $\Gamma(\cdot)$ is the gamma function.

Definition 2.3 The Liouville right-sided fractional derivative of order $\beta(t) > 0$ of a function $f : R^+ \to R$ on the half-axis R^+ is given by

$$(D_-^{\beta(t)}f)(t) = (-1)^{\lceil\beta(t)\rceil} \frac{d^{\lceil\beta(t)\rceil}}{dt^{\lceil\beta(t)\rceil}} (I_-^{\lceil\beta(t)\rceil-\beta(t)}f)(t) \quad (2.2)$$

where $\lceil\beta(t)\rceil := \min\{x \in I : x \geq \beta(t)\}$, provided that the right side is pointwise defined on R^+.

Lemma 2.1 Let Y be a solution of (1.3) and $H(t) = \int_t^\infty (z-t)^{-\beta(t)} Y(z)\,dz$, (2.3)

Then $H'(t) = -\Gamma_{(1-\beta(t))}(D_-^{\beta(t)} Y)(t)$, for $0 < \beta(t) < 1$, $t > 0$ (2.4)

Lemma 2.2. If U and V are positive, then $mUV^{m-1} - U^m \leq (m-1)V^m$, $m > 1$ (2.5)

3. Oscillation Results

Theorem 3.1. Suppose that (A_1) – (A_3) and $\int_{t_0}^\infty g^{-1}\left(\frac{1}{a(z)}\right)dz = \infty$ (3.1)

hold. Moreover, assume that there exists a positive function $\gamma \in C^1[t_0, \infty)$ such that

$$\lim_{t\to\infty} \sup \int_{t_0}^t \left[\ell_1 \gamma(z) b(z) - \frac{a(z)(\gamma'(z))^2}{4\ell_2 \Gamma_{(1-\beta(t))}\,\gamma(z)}\right]dz = \infty \quad (3.2)$$

where ℓ_1, ℓ_2 are defined as in (A_3). Then every solution of (1.3) is oscillatory.

Proof. Assume that Y is a nonoscillatory solution of (1.3). Without loss of generality,

We can assume that Y is an eventually positive solution of (1.3). Then there exists $t_1 \in [t_0, \infty)$ such that

$$Y(t) > 0,\ H(t) > o \text{ for } t \in [t_1, \infty) \quad (3.3)$$

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)
https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

where H is defined as in (2.3). Therefore, it follows from (1.3) that $[a(t)\, g(D_{-}^{\beta(t)}Y)(t)]' = b(t)\, f(H(t)) > 0$ for $t \in [t_1, \infty)$ (3.4)

Thus, $a(t)\, g(D_{-}^{\beta(t)}Y)(t)$ is strictly increasing on $[t_1, \infty)$ and is eventually of one sign. Since $a(t) > 0$ for $t \in [t_0, \infty)$ and (A_2), we see that $(D_{-}^{\beta(t)}Y)(t)$ is eventually of one sign. We now claim that $(D_{-}^{\beta(t)}Y)(t) < 0$ for $t \in [t_1, \infty)$ (3.5)

If not, then $(D_{-}^{\beta(t)}Y)(t)$ is eventually positive, and there exists $t_2 \in [t_1, \infty)$ such that $(D_{-}^{\beta(t)}Y)(t) > 0$ .Since $a(t)\, g(D_{-}^{\beta(t)}Y)(t)$ is strictly increasing on $[t_1, \infty)$, it is clear that $a(t)\, g(D_{-}^{\beta(t)}Y)(t) \geq a(t_2)\, g(D_{-}^{\beta(t)}Y)(t_2) = d > 0$ for $t \in [t_2, \infty)$.

From (2.4), we have

$$-\frac{H'(t)}{\Gamma_{(1-\beta(t))}} = (D_{-}^{\beta(t)}Y)(t)$$

$$\geq g^{-1}\left(\frac{d}{a(t)}\right)$$

$$\geq \alpha\, g^{-1}(d)\, g^{-1}\left(\frac{1}{a(t)}\right) \text{ for } t \in [t_2, \infty). \quad (3.6)$$

We get $g^{-1}\left(\frac{1}{a(t)}\right) \leq -\frac{H'(t)}{\alpha\, g^{-1}(d)\, \Gamma_{(1-\beta(t))}}$, for $t \in [t_2, \infty)$. (3.7)

Integrating the above inequality from t_2 to t, we have

$$\int_{t_2}^{t} g^{-1}\left(\frac{1}{a(s)}\right) ds \leq -\frac{H(t) - H(t_2)}{\alpha\, g^{-1}(d)\, \Gamma_{(1-\beta(t))}}$$

$$< \frac{H(t_2)}{\alpha\, g^{-1}(d)\, \Gamma_{(1-\beta(t))}}, \text{ for } t \in [t_2, \infty). \quad (3.8)$$

Let $t \to \infty$, we see $\int_{t_2}^{\infty} g^{-1}\left(\frac{1}{a(z)}\right) dz \leq \frac{H(t_2)}{\alpha\, g^{-1}(d)\, \Gamma_{(1-\beta(t))}} < \infty$. (3.9)

This contradicts (3.1). Hence (3.5) holds.

Define the function w by the Riccati substitution

$$w(t) = \gamma(t)\, \frac{-\, a(t)\, g(D_{-}^{\beta(t)}Y)(t)}{H(t)}, \text{ for } t \in [t_1, \infty). \quad (3.10)$$

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)

https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

We have $w(t) > 0$ for $t \in [t_1, \infty)$. From (3.10), (1.3), (2.4) and (A_1)–(A_3), it follows that

$$w'(t) = \frac{\gamma(t)}{H(t)}[-a(t)\, g(D_-^{\beta(t)}Y)(t)]' + \left(\frac{\gamma(t)}{H(t)}\right)'[-a(t)\, g\left(D_-^{\beta(t)}Y\right)(t)]$$

$$= -\gamma(t)\, b(t)\frac{f(H(t))}{H(t)} + \frac{\gamma'(t)H(t)-\gamma(t)H'(t)}{H^2(t)}[-a(t)\, g\left(D_-^{\beta(t)}Y\right)(t)]$$

$$= -\gamma(t)\, b(t)\frac{f(H(t))}{H(t)} + \frac{\gamma'(t)}{\gamma(t)}\, w(t) - \frac{H'(t)}{H(t)}\, w(t)$$

$$= -\gamma(t)\, b(t)\frac{f(H(t))}{H(t)} + \frac{\gamma'(t)}{\gamma(t)}\, w(t) - \frac{\Gamma_{(1-\beta(t))}w^2(t)}{\gamma(t)a(t)}\frac{(D_-^{\beta(t)}Y)(t)}{g(D_-^{\beta(t)}Y)(t)}$$

$$w'(t) \quad \le -\ell_1\, \gamma(t)\, b(t) + \frac{\gamma'(t)}{\gamma(t)}\, w(t) - \frac{\ell_2\Gamma_{(1-\beta(t))}}{\gamma(t)a(t)}w^2(t). \tag{3.11}$$

Let $U(t) = \sqrt{\frac{\ell_2\Gamma_{(1-\beta(t))}}{\gamma(t)a(t)}}\, w(t)$, $V(t) = \frac{1}{2}\sqrt{\frac{\gamma(t)a(t)}{\ell_2\Gamma_{(1-\beta(t))}}}\,\frac{\gamma'(t)}{\gamma(t)}$ and $m = 2$

From Lemma 2.2 and (3.11) we get

$$w'(t) \quad \le -\ell_1\, \gamma(t)\, b(t) + \frac{a(t)\,(\gamma'(t))^2}{4\ell_2\Gamma_{(1-\beta(t))}\gamma(t)} \tag{3.12}$$

Integrating both sides of the inequality (3.12) from t_0 to t, we obtain

$$\infty > w(t_0) > w(t_0) - w(t)$$

$$\ge \int_{t_0}^{t}\left(\ell_1\, \gamma(z)b(z) - \frac{a(z)\,(\gamma'(z))^2}{4\ell_2\Gamma_{(1-\beta(t))}\gamma(z)}\right)dz \tag{3.13}$$

Taking the limit supremum of both sides of the above inequality as $t \to \infty$, we get

$$\limsup_{t\to\infty} \int_{t_0}^{t}\left(\ell_1\, \gamma(z)b(z) - \frac{a(z)\,(\gamma'(z))^2}{4\ell_2\Gamma_{(1-\beta(t))}\gamma(z)}\right)dz \tag{3.14}$$

$$< w(t_0) < \infty$$

which contradicts (3.2). The proof is complete.

Theorem 3.2. Suppose that (A_1) – (A_3) and $\int_{t_0}^{\infty} g^{-1}\left(\frac{1}{a(z)}\right)dz < \infty$ (3.15)

hold, g is an increasing function, and that there exists a positive function $\gamma \in C^1[t_0, \infty)$ such that (3.2) holds.

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)
https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

Furthermore, assume that for every constant $T \geq t_0$, $\int_T^{\infty} g^{-1}\left(\frac{1}{a(t)}\int_T^t b(z)dz\right) dt = \infty$. (3.16)

Then every solution Y of (1.3) is oscillatory or satisfies $\lim_{t\to\infty} \int_{t_0}^{\infty}(z-t)^{-\beta(t)}Y(z)\,dz = 0$.

Proof. Suppose that Y is a nonoscillatory solution of (1.3). Without loss of generality, assume that Y is an eventually positive solution of (1.3). Proceeding as in the proof of Theorem 3.1, there are two cases for the sign of $(D_-^{\beta(t)}Y)(t)$. The proof when $(D_-^{\beta(t)}Y)(t)$ is eventually negative is similar to that of Theorem 3.1 and hence is omitted.

Assume that $(D_-^{\beta(t)}Y)(t)$ is eventually positive. Then there exists $t_2 \geq t_1$ such that $(D_-^{\beta(t)}Y)(t) > 0$ for $t \geq t_2$.

From (2.4), we get $H'(t) < 0$ for $t \geq t_2$. We get $\lim_{t\to\infty} H(t) = N \geq 0$ and $H(t) \geq N$. We claim that N= 0.

Assume not, i.e., $N > 0$, then from (A_3) we get

$$[a(t)\, g(D_-^{\beta(t)}Y)(t)]' = b(t)\, f(H(t)) \geq \ell_1 N\, b(t), \text{ for } t \in [t_2, \infty] \qquad (3.17)$$

Integrating both sides of the last inequality from t_2 to t, we have

$$a(t)\, g(D_-^{\beta(t)}Y)(t) \geq a(t_2)\left(g(D_-^{\beta(t)}Y)(t_2)\right) + \ell_1 N \int_{t_2}^t b(z)dz$$

$$> \ell_1 N \int_{t_2}^t b(z)dz, \text{ for } t \in [t_2, \infty]. \qquad (3.18)$$

Hence, from (2.4) & (A_2) , we get

$$-\frac{H'(t)}{\Gamma(1-\beta(t))} = (D_-^{\beta(t)}Y)(t) \geq g^{-1}\left(\frac{\ell_1 N \int_{t_2}^t b(z)\,dz}{a(t)}\right)$$

$$> \alpha\, g^{-1}(\ell_1 N)\, g^{-1}\left(\frac{\int_{t_2}^t b(z)\,dz}{a(t)}\right) \text{ for } t \in [t_2, \infty]. \qquad (3.19)$$

Integrating both sides of the last inequality from t_2 to t, we obtain

$$H(t) \leq H(t_2) - \alpha\, g^{-1}(\ell_1 N)\ \Gamma_{(1-\beta(t))} \int_{t_2}^t g^{-1}\left(\frac{\int_{t_2}^v b(z)\,dz}{a(v)}\right) dv, \text{ for } t \in (t_2, \infty). \qquad (3.20)$$

Musik in Bayern

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)

https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

Letting $t \to \infty$, from (3.16), we get $\lim_{t\to\infty} H(t) = -\infty$. This contradicts $H(t) > 0$. Therefore, we have $N = 0$, ie, $\lim_{t\to\infty} H(t) = 0$.

In view of (2.3), we see that the proof is complete.

4. Example

Example 4.1. Consider the variable order fractional differential equation

$$\left[t^{1/3} D_{-}^{t/2} Y)(t)\right]' - t\left(\int_t^{\infty}(z-t)^{-t/2}\, Y(z)\, dz\right) = 0, \quad \text{for } t > 0. \tag{4.1}$$

In (4.1), $\beta(t) = \frac{t}{2}$, where $0 < t < 2$, $a(t) = t^{1/3}$, $b(t) = t$, and $f(u) = g(u) = u$. Take $t_0 = 1$, $\ell_1 = \ell_2 = 1$. It is clear that conditions (A_1)–(A_3) and (3.1) hold.

Furthermore, taking $\gamma(t) = t$, we have

$$\limsup_{t\to\infty} \int_{t_0}^{t}\left[\ell_1\gamma(z)\, b(z) - \frac{a(z)(\gamma'(z))^2}{4\ell_2\Gamma_{(1-\beta(t))}\, \gamma(z)}\right] ds = \limsup_{t\to\infty} \int_1^t \left[z^2 - \frac{z^{1/3}}{4\, z\, \Gamma_{(1-\frac{t}{2})}}\right] dz = \infty,$$

which shows that (3.2) holds. Therefore, by Theorem 3.1 every solution of (4.1) is oscillatory.

Example 4.2. Consider the variable order fractional differential equation

$$\left[t^{3/2} D_{-}^{t/2} Y)(t)\right]' - t\left(\int_t^{\infty}(z-t)^{-t/2}\, Y(z)\, dz\right) = 0, \text{ for } t > 0. \tag{4.2}$$

In (4.2), $\beta(t) = \frac{t}{2}$, where $0 < t < 2$, $a(t) = t^{3/2}$, $b(t) = t$, and $f(u) = g(u) = u$. Take $t_0 = 1$, $\ell_1 = \ell_2 = 1$. It is clear that conditions (A_1)–(A_3) and (3.15) hold.

Taking $\gamma(t) = t$, we have

$$\limsup_{t\to\infty} \int_{t_0}^{t}\left[\ell_1\gamma(z)\, b(z) - \frac{a(z)(\gamma'(z))^2}{4\ell_2\Gamma_{(1-\beta(t))}\, \gamma(z)}\right] dz = \limsup_{t\to\infty} \int_1^t \left(z^2 - \frac{z^{3/2}}{4\, z\, \Gamma_{(1-\frac{t}{2})}}\right) dz = \infty.$$

which shows that (3.2) holds.

Furthermore, for every constant $T \geq 1$, we have

$$\int_T^{\infty} g^{-1}\left(\frac{1}{a(t)} \int_T^t b(z)\, dz\right) dt = \int_T^{\infty}\left(\frac{1}{t^{3/2}} \int_T^t z\, dz\right) dt = \infty.$$

which shows that (3.16) holds. Therefore, by Theorem 3.2 every solution of (4.2) is oscillatory or satisfies $\lim_{t\to\infty} \int_{t_0}^{\infty}(z-t)^{-\beta(t)}\, Y(z)\, dz = 0$.

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)

https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512

5. Conclusion

This article provides the oscillation criteria of variable order nonlinear fractional differential equation together with examples. The conclusion is that if the conditions (3.1),(3.2),(3.15) and (3.16) are satisfied, then each solution of the equation (1.3) oscillates. In further research, we aim to achieve the intended outcome for the oscillatory behavior of a class of mixed fractional variable order differential equation.

References

[1] Oldham, KB, Spanier, J: The Fractional Calculus. Academic Press, San Diego (1974).

[2] Miller, KS, Ross, B: An Introduction to the Fractional Calculus and Fractional Differential Equations. John Wiley & Sons, New York (1993).

[3] Podlubny, I: Fractional Differential Equations. Academic Press, San Diego (1999).

[4] Kilbas, AA, Srivastava, HM, Trujillo, JJ: Theory and Applications of Fractional Differential Equations. Elsevier Science BV, Amsterdam (2006).

[5] Das, S: Functional Fractional Calculus for System Identification and Controls. Springer, New York (2008).

[6] Lakshmikantham, V.: Theory of fractional functional differential equations. Nonlinear Anal. 69, 3337–3343 (2008).

[7] Meng, Q, et al. Sufficient and necessary conditions for oscillation of linear fractional-order delay differential equations. Adv. Differ. Equ. (2021).

[8] Samko, SG, Kilbas, AA, Marichev OI: Fractional Integrals and Derivatives. Theory and Applications. Gordon and Breach Science Publishers, Yverdon, Switzerland (1993).

[9]. Grace, SR, Agarwal, RP, Wong, PJY, Zafer A.: On the oscillation of fractional differential equations. Fractional Calculus and Applied Analysis. (2012) 15, no. 2, 222–231.

10] Chen, D-X: Oscillation criteria of fractional differential equations. Adv. Differ. Equ. (2012) 2012, no. article 33, 1–18.

[11] Marian, SL, Sagayaraj, MR, Selvam, AGM, Loganathan, MP: Oscillation of fractional nonlinear difference equations.
Math. Aeterna. (2012) 2, no. 9-10, 805–813.

[12] Prakash, P, Harikrishnan S, Benchohra MI: Oscillation of certain nonlinear fractional partial differential equation with damping term.
Appl. Math. Lett. (2015), 43, 72–79.

[13] Zheng, B, Feng, Q: Some new oscillation criteria for a class of nonlinear fractional differential equations with damping term.
J. Appl. Math. (2013), 2013, 912072.

[14] Zheng, B: Oscillation for a class of nonlinear fractional differential equations with damping term. J. Adv. Math. Stud. (2013), 6, 107–115.

[15] Xu, R: Oscillation criteria for nonlinear fractional differential equations. J. Appl. Math. (2013), 2013, 971357.

[16] Pavithra, S, Muthulakshmi V: Oscillatory behavior for a class of fractional differential equations. Int. J. Pure Appl. Math. (2017), 115, 93–107.

[17] Somia, D, Brahim, N: A new class of mixed fractional differential equations with integral boundary conditions.
Moroc. J. Pure Appl. Anal. (2021), 7, 227–247.

[18] Chen, D: Oscillatory behavior of a class of fractional differential equations with damping. Universitatea Politehnica din Bucuresti
Scientific Bulletin, Series A. (2013), 75, no. 1, 107–118.

[19] Zhao, Z, Sun, Y, Zhang, C: Oscillation for a class of fractional differential equation.
Discrete Dynamics in Nature and Society (2013) 2013.

[20] Bai, Z, Lü,H: Positive solutions for boundary value problem of nonlinear fractional differential equation.
J. Math. Anal. Appl. (2005), 311, no. 2, 495–505.

ISSN: 0937-583x Volume 90, Issue 12 (Dec -2025)

https://musikinbayern.com DOI https://doi.org/10.15463/gfbm-mib-2025-512